Energy Conversion – Exam II

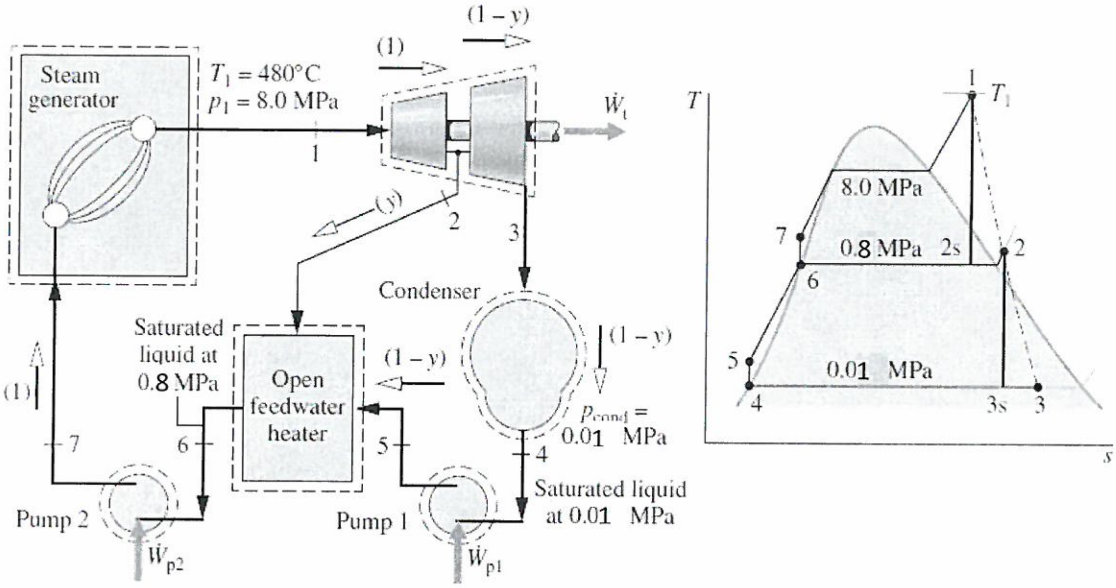
School of Engineering – Dep. of Industrial & Mechanical Eng.

Name: Soluti° Key
Date: Monday, May 19th 2014; 05:00 PM
Location: ENG Auditorium
Instructor: Dr. Wassim Habchi
Notes: No documents allowed
Time: 2 hours

[/100]

Problem I (25 points)

Consider a regenerative vapor power cycle with one open feedwater heater as shown below:



Steam enters the turbine at 8.0MPa, 480°C and expands to 0.8MPa, where some of the steam is extracted and diverted to the open feedwater heater operating at 0.8MPa. The remaining steam expands through the second-stage turbine to the condenser pressure of 0.01MPa. Saturated liquid exits the open feedwater heater at 0.8MPa. The isentropic efficiency of each turbine stage is 85% and each pump operates isentropically. If the net power output of the cycle is 100MW, determine:

a) The fraction y of steam extracted from the turbine to the feedwater heater (**10 points**).

b) The thermal efficiency of this cycle (**10 points**).

c) The mass flow rate of steam entering the first turbine stage **in tons/h** (**5 points**).

Solution:

a) Energy balance on open FWH:

$$y h_2 + (1-y) h_5 = h_6 \implies y(h_2 - h_5) = h_6 - h_5$$

$$\implies \boxed{y = \frac{h_6 - h_5}{h_2 - h_5}}$$

but $\boxed{h_1 = 3349.02\ KJ/Kg}$ and $s_1 = 6.65912\ KJ/Kg$

@ (2s): $\left.\begin{array}{l} P_{2s} = 0.8MPa \\ s_{2s} = s_1 \end{array}\right\} \longrightarrow$ $x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.65912 - 2.0457}{4.6160}$

$x_{2s} = 0.99944$

$\Rightarrow h_{2s} = h_f + x_{2s} h_{fg} = 720.87 + 0.99944 \times 2047.5 = 2767.22 KJ/Kg$

but $\frac{h_1 - h_{2a}}{h_1 - h_{2s}} = 0.85 \longrightarrow h_{2a} = h_1 - 0.85(h_1 - h_{2s})$

$= 3349.02 - 0.85(3349.02 - 2767.2$

$\boxed{h_{2a} = 2854.49\ KJ/Kg}$

* $h_6 = h_f @ 0.8MPa = \boxed{720.87\ KJ/Kg}$

* $h_5 = h_4 + v_4(P_5 - P_4) = h_f @ 0.01MPa + v_f @ 0.01MPa (P_5 - P_4)$

$= 191.81 + 0.001010(0.8 - 0.01) \times 10^3$

$\boxed{h_5 = 192.61\ KJ/Kg}$

$\Rightarrow y = \frac{720.87 - 192.61}{2854.49 - 192.61} = \boxed{0.19845 \text{ or } 19.845\%}$

b) $\eta_{th} = \frac{\dot{W}_t - \dot{W}_{p_1} - \dot{W}_{p_2}}{\dot{Q}_{in}} = \frac{(h_1 - h_2) + (1-y)(h_2 - h_3) - (1-y)v_4(P_5 - P_4) - (P_7 - P_6)v_6}{h_1 - h_7}$

@ (2): $\left.\begin{array}{l} h_2 = 2854.49\ KJ/Kg \\ P_2 = 0.8 MPa \end{array}\right\} \longrightarrow s_2 = 6.84725\ KJ/Kg.K$

@ (3s): $\left.\begin{array}{l} P_{3s} = 0.01MPa = 10KPa \\ s_{3s} = s_2 = 6.84725\ KJ/Kg.K \end{array}\right\} \longrightarrow x_{3s} = \frac{s_{3s} - s_f}{s_{fg}} = \frac{6.84725 - 0.6492}{7.4996}$

$x_{3s} = 0.82645$

$\rightarrow h_{3s} = h_f + x_{3s} h_{fg} = 191.81 + 0.82645 \times 2392.1 = \boxed{2168.76\ KJ/Kg}$

but $\frac{h_2 - h_{3a}}{h_2 - h_{3s}} = 0.85 \longrightarrow h_{3a} = h_2 - 0.85(h_2 - h_{3s}) = 2854.49 - 0.85(2854.49 - 2168.76)$

$\Rightarrow \boxed{h_{3a} = 2271.62 \text{ KJ/Kg}}$

$v_6 = v_f @ 0.8\text{MPa} = \boxed{0.001115 \text{ m}^3/\text{Kg}}$

and $h_7 = h_6 + v_6 (P_7 - P_6) = 720.87 + 0.001115(8-0.8)\times10^3$

$\Rightarrow \boxed{h_7 = 728.9 \text{ KJ/Kg}}$

$$\Rightarrow \eta_{th} = \frac{(3349.02 - 2854.49) + (1-0.19845)(2854.49 - 2271.62) - (1-0.19845)(0.00101)(8000-10) - 0.001115(8000-800)}{3349.02 - 728.9}$$

$$\Rightarrow \eta_{th} = \frac{(494.56 + 467.2) - 0.63956 - 8.028}{2620.12} = \boxed{0.36376 \simeq 36.4\%}$$

2nd method:

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{(1-y)(h_3 - h_4)}{h_1 - h_7} = 1 - \frac{(1-0.19845)(2271.62 - 191.81)}{3349.02 - 728.9}$$

$= 0.36374 \simeq \boxed{36.4\%}$

c) $\dot{W}_{net,out} = 100\text{MW} = \dot{m}\, q_{net,in} = \dot{m}(q_{in} - q_{out})$

$$\Rightarrow \dot{m} = \frac{100\times10^3}{(3349.02 - 728.9) - (1-0.19845)(2271.62 - 191.81)}$$

$\dot{m} = 104.93 \text{ Kg/s} = 104.93 \times 10^{-3} \times 3600 \text{ tons/h}$

$\Rightarrow \boxed{\dot{m} = 377.75 \text{ tons/h}}$

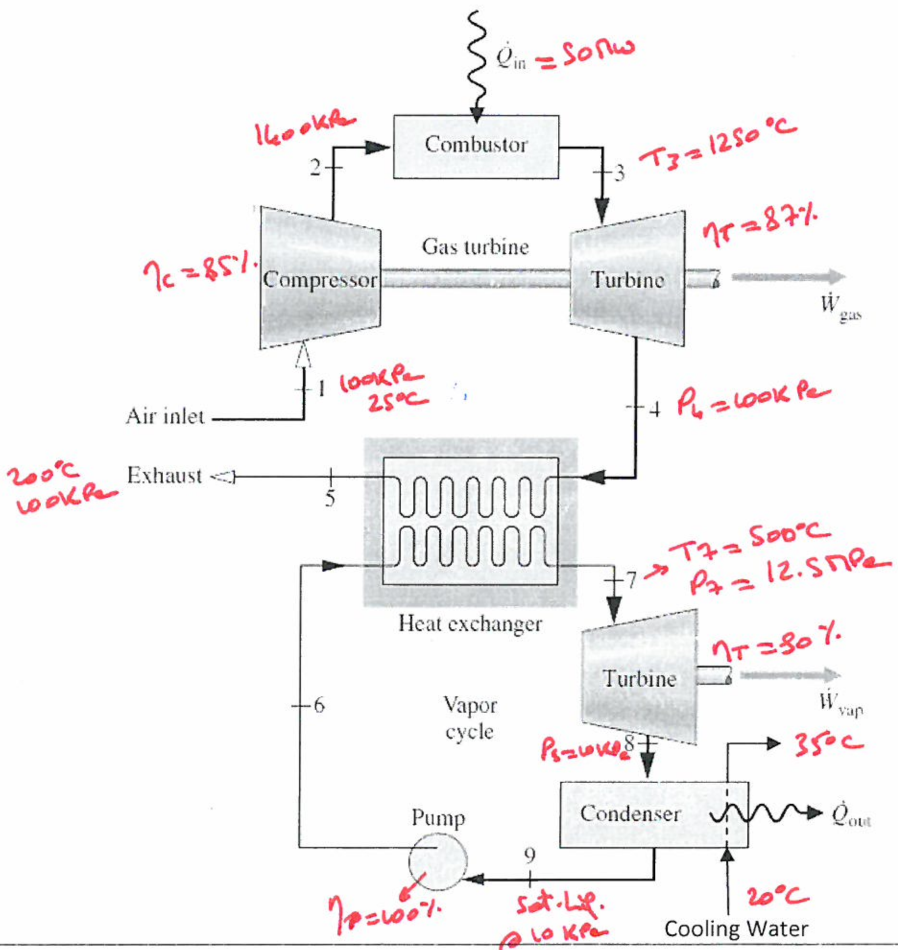
Problem II (30 points)

Air enters the compressor of a combined gas turbine-vapor power plant at 100 kPa, 25°C. The isentropic compressor efficiency is 85% and the compressor pressure ratio is 14. The air passing through the combustor receives energy by heat transfer at a rate of 50 MW with no significant decrease in pressure. At the inlet to the turbine the air is at 1250°C. The air expands through the turbine, which has an isentropic efficiency of 87%, to a pressure of 100kPa. Then, the air passes through the interconnecting heat exchanger and is finally discharged at 200°C, 100kPa. Steam enters the turbine of the vapor cycle at 12.5MPa, 500°C, and expands to a condenser pressure of 10kPa. Water enters the pump as a saturated liquid at 10kPa. The turbine and pump have isentropic efficiencies of 90 and 100%, respectively. Cooling water enters the condenser at 20°C and exits at 35°C. Determine:

a) The mass flow rates of the air, steam, and cooling water, each in kg/s (**10 points**). (given $C_w = 4.2$ kJ/kg·K)

b) The net power developed by the gas turbine cycle and the vapor cycle, respectively, each in MW (**10 points**).

c) The thermal efficiency of the combined cycle (**10 points**).



Solution:

a) * Energy balance on combustor:

$\dot{Q}_{in} = \dot{m}_{air}(h_3 - h_2) \longrightarrow \boxed{\dot{m}_{air} = \dfrac{\dot{Q}_{in}}{h_3 - h_2}}$

A-17: $h_3 = h@1523K = \boxed{1663.872 \text{ kJ/kg}}$ & $P_{r3} = 641.945$

$\frac{P_{r_2}}{P_{r_1}} = \frac{P_2}{P_1} = 14$ but $P_{r_1} = P_r @ 288K = 1.3543$

$\Rightarrow P_{r_2} = 14 \times 1.3543 = 18.9602 \Rightarrow$ $\boxed{h_{2s} = 632.35 \text{ KJ/Kg.}}$

but $\frac{h_1 - h_{2s}}{h_1 - h_{2a}} = \eta_c \Rightarrow h_{2a} = h_1 - \frac{h_1 - h_{2s}}{\eta_c} = 288.18 - \frac{288.18 - 632.35}{0.85}$

$\boxed{h_{2a} = 693.32 \text{ KJ/Kg}}$

$\Rightarrow \dot{m}_{air} = \frac{50 \times 10^3}{1663.872 - 693.32} = \boxed{51.41 \text{ Kg/s}}$

* Energy balance on Heat exchanger:

$\dot{m}_{air}(h_4 - h_5) = \dot{m}_s(h_7 - h_6)$

$\Rightarrow \boxed{\dot{m}_s = \dot{m}_{air} \frac{h_4 - h_5}{h_7 - h_6}}$

$\frac{P_{r_4}}{P_{r_3}} = \frac{P_4}{P_3} = \frac{1}{14} \longrightarrow P_{r_4} = \frac{P_{r_3}}{14} = \frac{641.945}{14} = 45.8532$

A-17 $\Rightarrow h_{4s} = 812.5$ KJ/Kg

but $\frac{h_3 - h_{4a}}{h_3 - h_{4s}} = 0.87 \longrightarrow h_{4a} = h_3 - 0.87(h_3 - h_{4s})$

$= 1663.872 - 0.87(1663.872 - 812.5)$

$\Rightarrow \boxed{h_{4a} = 923.18 \text{ KJ/Kg}}$

@⑤: $h_5 = h @ 473K = \boxed{475.315 \text{ KJ/Kg}}$

@⑦: $\boxed{h_7 = 3343.6 \text{ KJ/Kg}}$ and $S_7 = 6.4651$ KJ/Kg.K

and $h_6 = h_8 + v_8(P_6 - P_8) = h_{f@60kPa} + v_{f@60kPa}(P_6 - P_8)$

$= 151.81 + 0.00101(12500 - 60) = \boxed{204.425 \text{ KJ/Kg}}$

$\Rightarrow \dot{m}_s = 51.41 \times \frac{923.18 - 475.315}{3343.6 - 204.425} = \boxed{7.335 \text{ Kg/s}}$

* Energy balance on condenser:

$$\dot{m}_w C_w \Delta T_w = \dot{m}_s (h_8 - h_5)$$ but $h_5 = h_f@10kPa = 191.81\ kJ/kg$

$$\dot{m}_w = \frac{\dot{m}_s (h_8 - h_5)}{C_w \Delta T_w}$$

@(8s): $P_{8s} = 10 kPa$, $S_{8s} = S_7 = 6.4651\ kJ/kg.K$ $\rightarrow x_{8s} = \frac{S_{8s} - S_f}{S_{fg}} = \frac{6.4651 - 0.6492}{7.4996} = 0.7755$

$$\Rightarrow h_{8s} = h_f + x_{8s} h_{fg} = 191.81 + 0.7755 \times 2392.1 = 2046.88\ kJ/kg$$

but $\frac{h_7 - h_{8a}}{h_7 - h_{8s}} = 0.8 \Rightarrow h_{8a} = h_7 - 0.8(h_7 - h_{8s})$

$$= 3343.6 - 0.8(3343.6 - 2046.88)$$

$$\boxed{h_{8a} = 2176.55\ kJ/kg}$$

$$\Rightarrow \dot{m}_w = 7.335 \times \frac{2176.55 - 191.81}{4.2 \times 15} = \boxed{231.1\ kg/s}$$

b) * $\dot{W}_{net,gas} = \dot{m}_{air}[(h_3 - h_4) - (h_2 - h_1)] = 51.41[(1663.872 - 923.18) - (631.32 - 298.18)]$

$\dot{W}_{net,gas} = 17867.65\ kW \simeq \boxed{17.87\ MW}$

* $\dot{W}_{net,vap} = \dot{m}_s[(h_7 - h_8) - (h_6 - h_5)] = 7.335[(3343.6 - 2176.55) - (204.425 - 191.81)]$

$\dot{W}_{net,vap} = 8467.78\ kW \simeq \boxed{8.47\ MW}$

c) $$\eta_{th} = \frac{\dot{W}_{net,gas} + \dot{W}_{net,vap}}{\dot{Q}_{in}} = \frac{17.87 + 8.47}{50} = 0.5268 = \boxed{52.68\%}$$

Problem III (25 points)

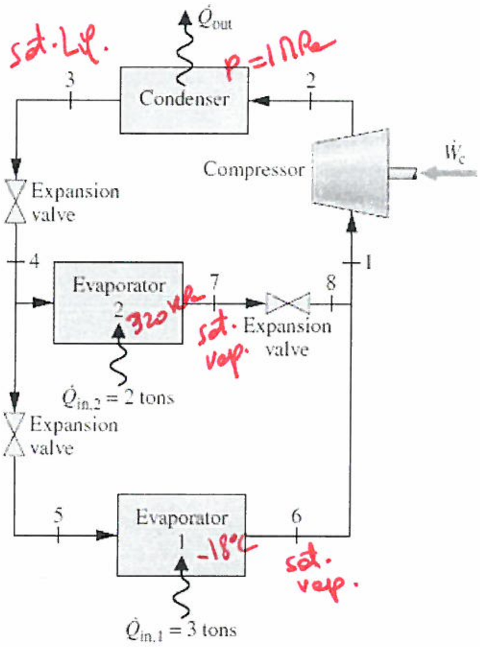
The figure below shows the schematic diagram of a vapor compression refrigeration system with two evaporators using Refrigerant 134a as the working fluid. This arrangement is used to achieve refrigeration at two different temperatures with a single compressor and a single condenser. The low-temperature evaporator operates at -18°C with saturated vapor at its exit and has a refrigerating capacity of 3 tons (1 ton of refrigeration is equivalent to 211kJ/min). The higher temperature evaporator produces saturated vapor at 320kPa at its exit and has a refrigerating capacity of 2 tons. Compression is isentropic to the condenser pressure of 1MPa. There are no significant pressure drops in the flows through the condenser and the two evaporators, and the refrigerant leaves the condenser as saturated liquid at 1MPa. Determine:

a) The mass flow rate of refrigerant through each evaporator, **in kg/min** (**10 points**).

b) The compressor power input, **in kW** (~~5~~ 8 **points**).

c) The rate of heat transfer from the refrigerant passing through the condenser, **in kW** (~~5~~ 4 **points**).

d) The Coefficient Of Performance of this refrigeration system (~~5~~ 3 **points**).



Solution:

a) $\dot{Q}_{in,1} = \dot{m}_1 (h_6 - h_5)$ but $h_5 = h_4 = h_3 = h_f @ 1MPa = 107.32$ kJ/kg

$h_6 = h_g @ -18°C = 238.64$ kJ/kg

$$\Rightarrow \dot{m}_1 = \frac{\dot{Q}_{in,1}}{h_6 - h_5} = \frac{3 \times 211}{238.64 - 107.32} = \boxed{4.784 \text{ kg/min}}$$

And $\dot{Q}_{in,2} = \dot{m}_2 (h_7 - h_4)$ with $h_7 = h_g @ 320kPa = 251.88$ kJ/kg

$$\Rightarrow \dot{m}_2 = \frac{\dot{Q}_{in,2}}{h_7 - h_4} = \frac{2 \times 211}{251.88 - 107.32} = \boxed{2.919 \text{ kg/min}}$$

b) $\dot{W}_c = (\dot{m}_1 + \dot{m}_2)(h_2 - h_1)$

but $\dot{m}_2 h_8 + \dot{m}_1 h_6 = (\dot{m}_1 + \dot{m}_2) h_1$ with $h_8 = h_7$

$\Rightarrow h_1 = \dfrac{\dot{m}_2 h_7 + \dot{m}_1 h_6}{\dot{m}_1 + \dot{m}_2} = \dfrac{2.319 \times 251.88 + 4.784 \times 239.64}{2.319 + 4.784}$

$\rightarrow$ $\boxed{h_1 = 244.27 \text{ kJ/kg}}$

and $P_1 = P_{sat}$ @ $-18°C = 144.69$ kPa.

$P = 0.14$ MPa & $h = 244.27$ kJ/kg $\rightarrow s = 0.96433$ kJ/kg·K

$\Rightarrow$ @ $P = 0.14469$ kP $\Rightarrow$ $\boxed{s_1 = 0.96207 \text{ kJ/kg·K}}$

$P = 0.18$ MPa & $h = 244.27$ kJ/kg $\rightarrow s = 0.94503$ kJ/kg·K

@ ②: $P_2 = 1$ MPa, $s_2 = s_1 = 0.96207$ kJ/kg·K $\longrightarrow$ $\boxed{h_2 = 285.87 \text{ kJ/kg}}$

$\Rightarrow \dot{W}_c = (2.319 + 4.784)(285.87 - 244.27) = 320.44$ kJ/min

$= \boxed{5.34 \text{ kW}}$

c) $\dot{Q}_{out} = (\dot{m}_1 + \dot{m}_2)(h_2 - h_3) = (2.319 + 4.784)(285.87 - 107.32)$

$= 1375.37$ kJ/min $= \boxed{22.92 \text{ kW}}$

d) $COP = \dfrac{\dot{Q}_{in,1} + \dot{Q}_{in,2}}{\dot{W}_c} = \dfrac{(2+3) \times 211}{320.44} = \boxed{3.29}$

Problem IV (20 points)

An ideal vapor-compression heat pump cycle with Refrigerant 134a as the working fluid provides heating at a rate of 15 kW to maintain a building at 20°C when the outside temperature is 5°C. Saturated vapor at 240kPa leaves the evaporator, and saturated liquid at 800kPa leaves the condenser. Determine:

a) The mass flow rate of refrigerant flowing through this cycle (**11 points**).

b) The power input to the compressor, **in kW** (**3 points**).

c) The coefficient of performance (**3 points**).

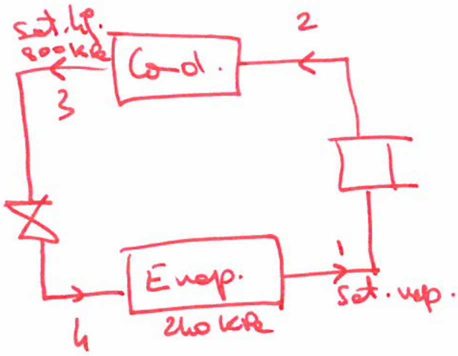
d) The maximum Coefficient Of Performance that this heat pump can reach (**3 points**).

Solution:

$\dot{Q}_H = 15 kW$ $T_H = 20°C$ $T_L = 5°C$

$P_{evap} = 240 kPa$

$P_{cond} = 800 kPa$



a) $\dot{Q}_H = \dot{m}(h_2 - h_3)$

$\Rightarrow \dot{m} = \dfrac{\dot{Q}_H}{h_2 - h_3}$ but $h_3 = h_f @ 800kPa = 95.47 kJ/kg$

And @①: $h_1 = h_g @ 240kPa = 247.28 kJ/kg$

$s_1 = s_g @ 240kPa = 0.93458 kJ/kg.K$

@②: $P_2 = 800 kPa$, $s_2 = s_1 = 0.93458 kJ/kg.K$ $\longrightarrow$ $h_2 = 272.31 kJ/kg$

$\Rightarrow \dot{m} = \dfrac{15}{272.31 - 95.47} = 0.0848 kg/s$

b) $\dot{W}_{comp} = \dot{m}(h_2 - h_1) = 0.0848 \times (272.31 - 247.28) = 2.12 kW$

c) $COP = \dfrac{\dot{Q}_H}{\dot{W}_{comp}} = \dfrac{15}{2.12} = 7.0754$

d) $COP_{carnot} = \dfrac{1}{1 - T_L/T_H} = \dfrac{1}{1 - 278/293} = 19.53$